Stereo Matching Using Belief Propagation Algorithm

ISL Lab Seminar

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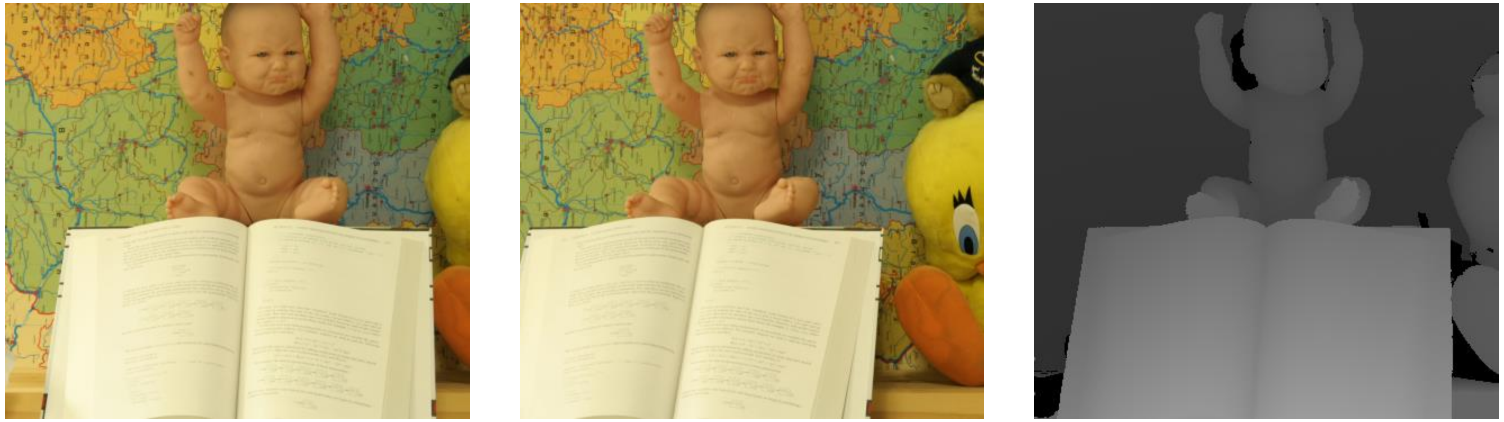
Contents

Introduction

❖ Stereo Vision

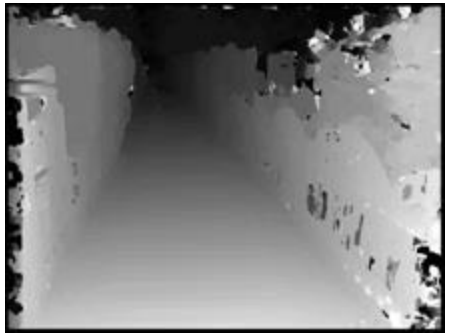
: Stereo vision is the extraction of 3D information from digital images, such as obtained by a CCD camera. **By comparing information about a scene from two vantage points**, 3D information can be extracted by examination of the relative positions of objects in the two panels. This is similar to the biological process **Stereopsis**.

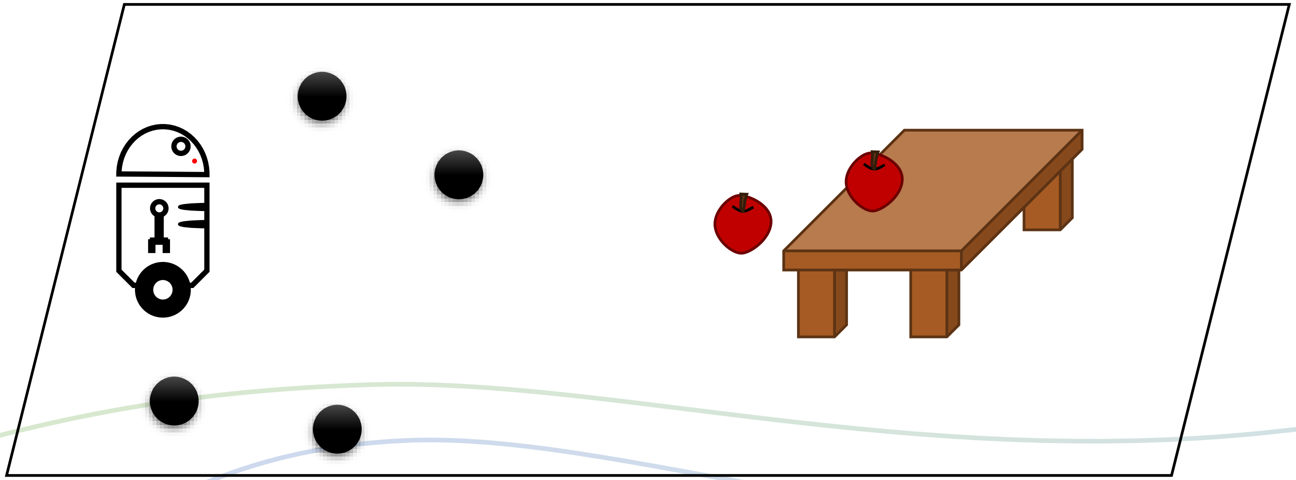


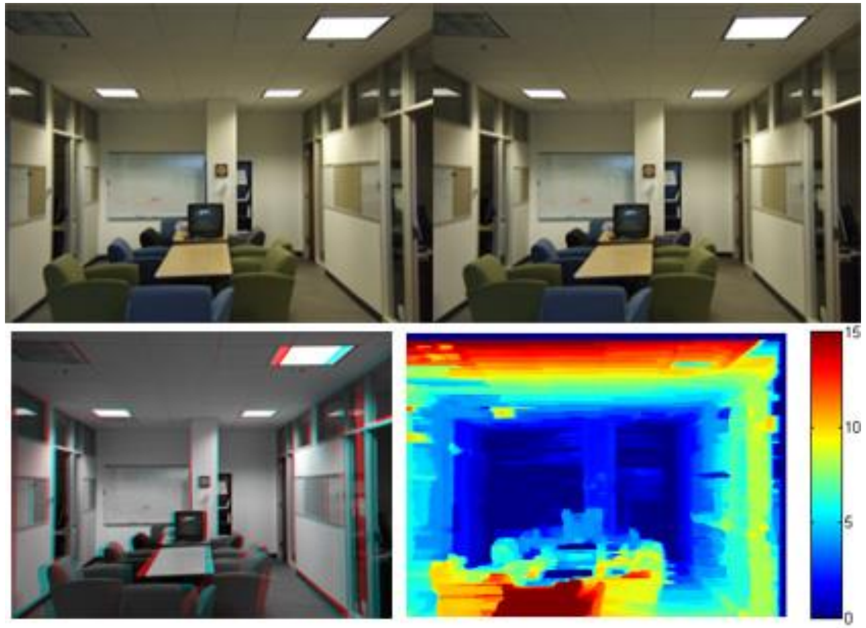
Introduction

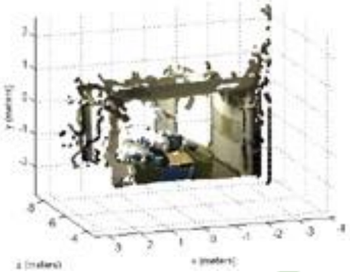
❖ Applications

: Stereo vision is highly important in fields such as robotics, to extract information about the relative position of 3D objects in the vicinity of autonomous systems.





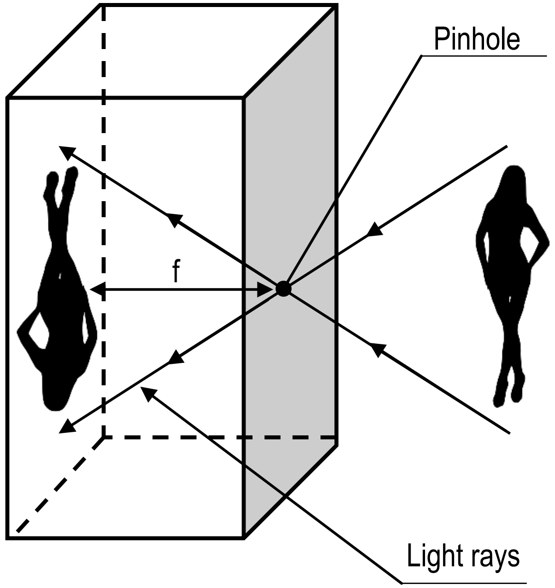


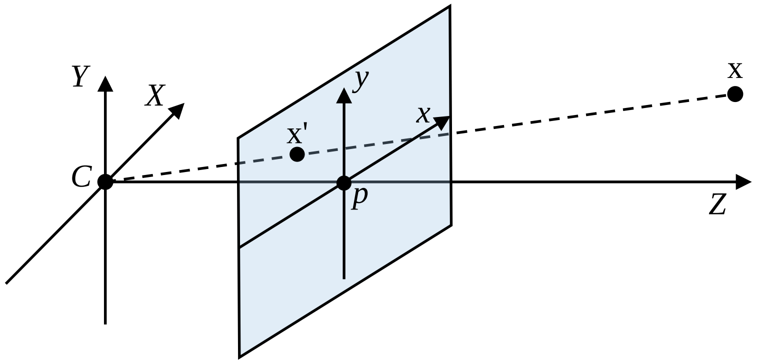


Camera Geometry

❖ Pin-hole camera model

: A simple camera without a lens

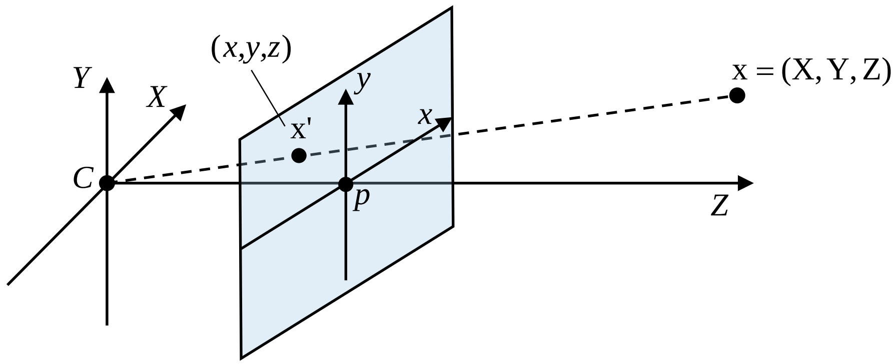


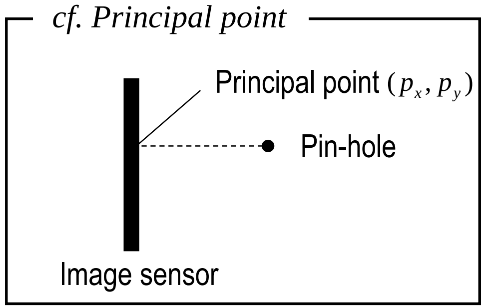


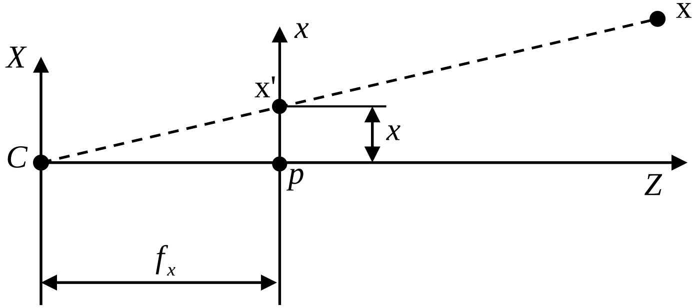
<Mathematical model>

Camera Geometry

❖ Pin-hole camera model







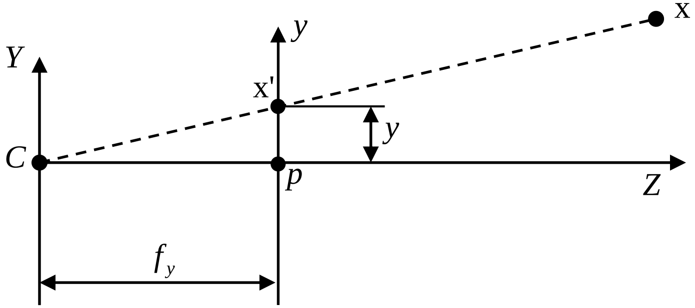
XZ plane

$$f_x : Z = x : X$$

$$f_x X = Zx$$

$$x = \frac{f_x X}{Z}$$

$$= \frac{f_x X}{Z} + p_x$$



YZ plane

$$f_y : Z = y : Y$$

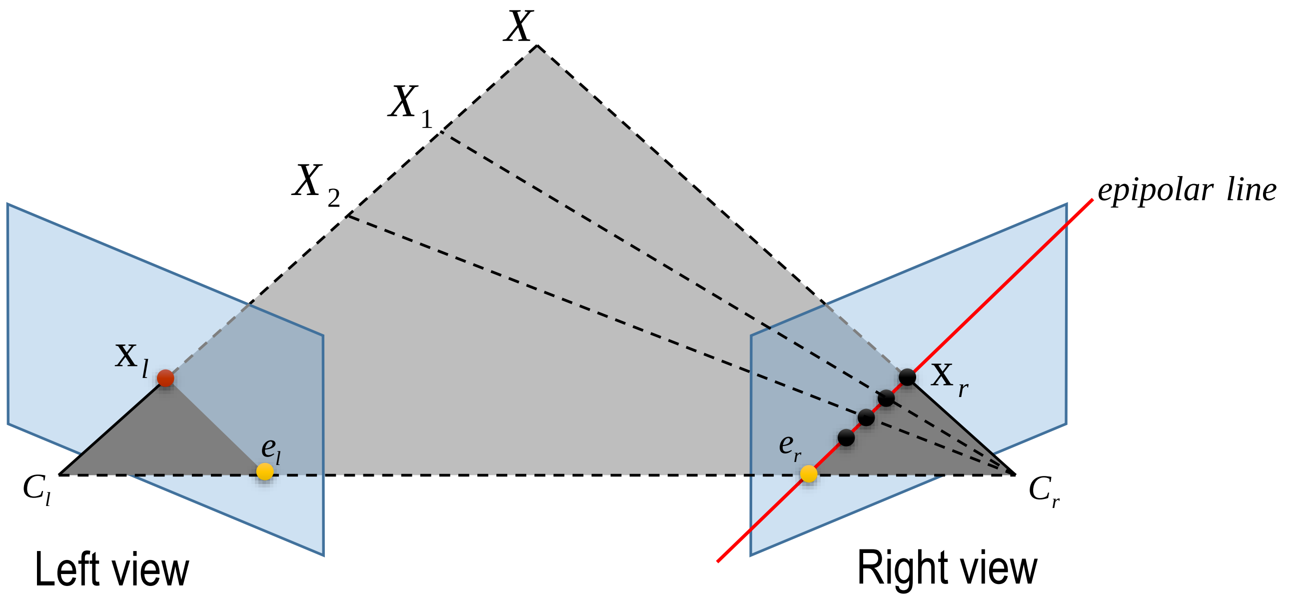
$$f_y Y = Zy$$

$$y = \frac{f_y Y}{Z}$$

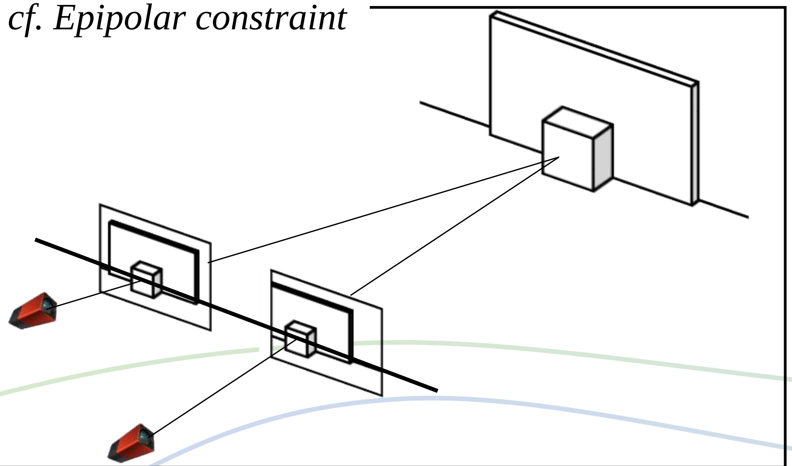
$$= \frac{f_y Y}{Z} + p_y$$

Camera Geometry

❖ Epipolar geometry

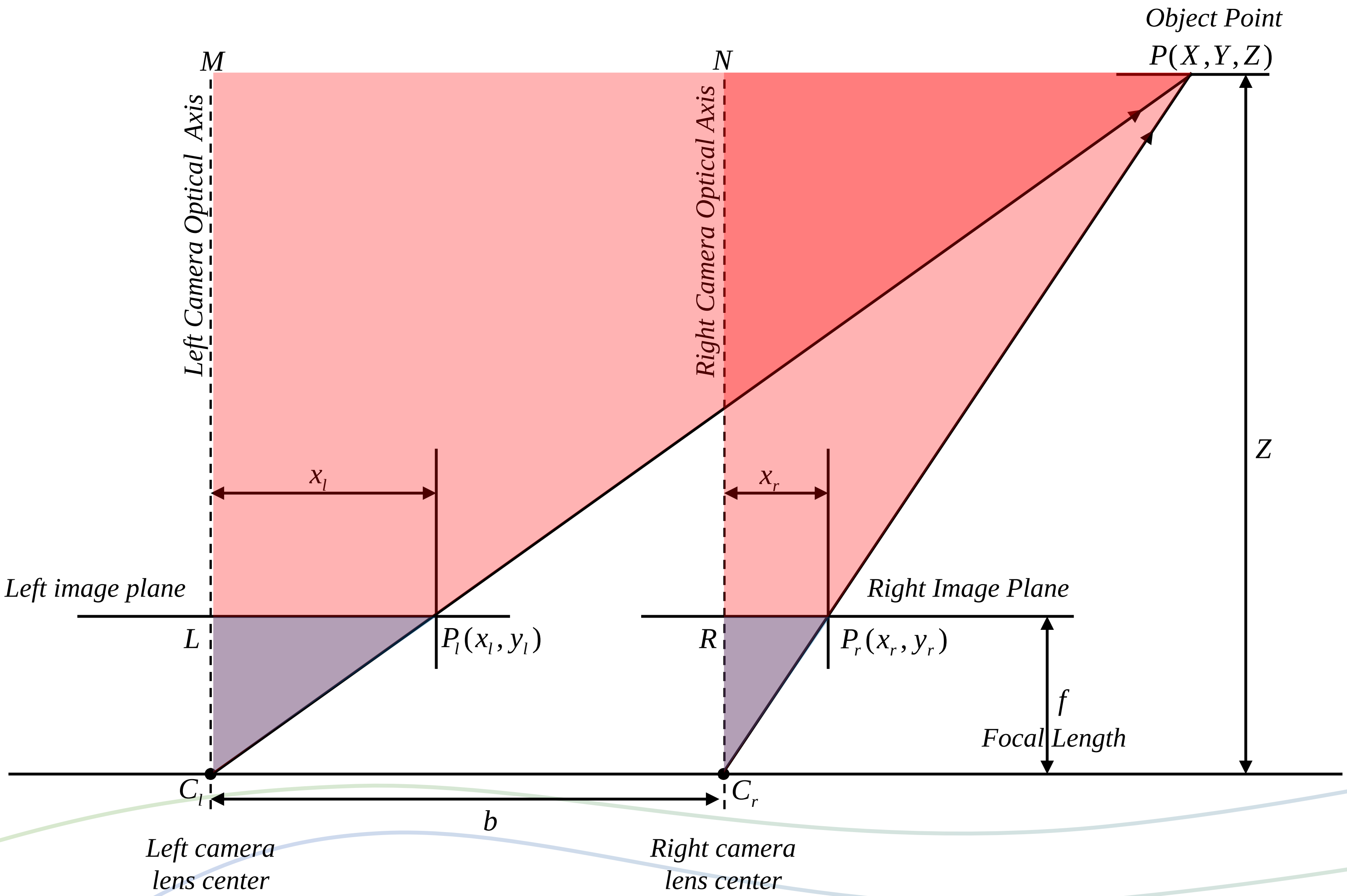


cf. Epipolar constraint



Stereo Matching

❖ Stereo Camera model



$$\Delta PMC_l \overset{similar}{\longleftrightarrow} \Delta P_l L C_l$$

$$\frac{X}{Z} = \frac{x_l}{f} \quad \text{———— a}$$

$$\Delta PNC_r \overset{similar}{\longleftrightarrow} \Delta P_r R C_r$$

$$\frac{X-b}{Z} = \frac{x_r}{f} \quad \text{———— b}$$

from a $X = \frac{x_l}{f} Z$ from b $X = \frac{x_r}{f} Z + b$

$$\frac{x_l}{f} Z = \frac{x_r}{f} Z + b, \quad \frac{x_l - x_r}{f} Z = b$$

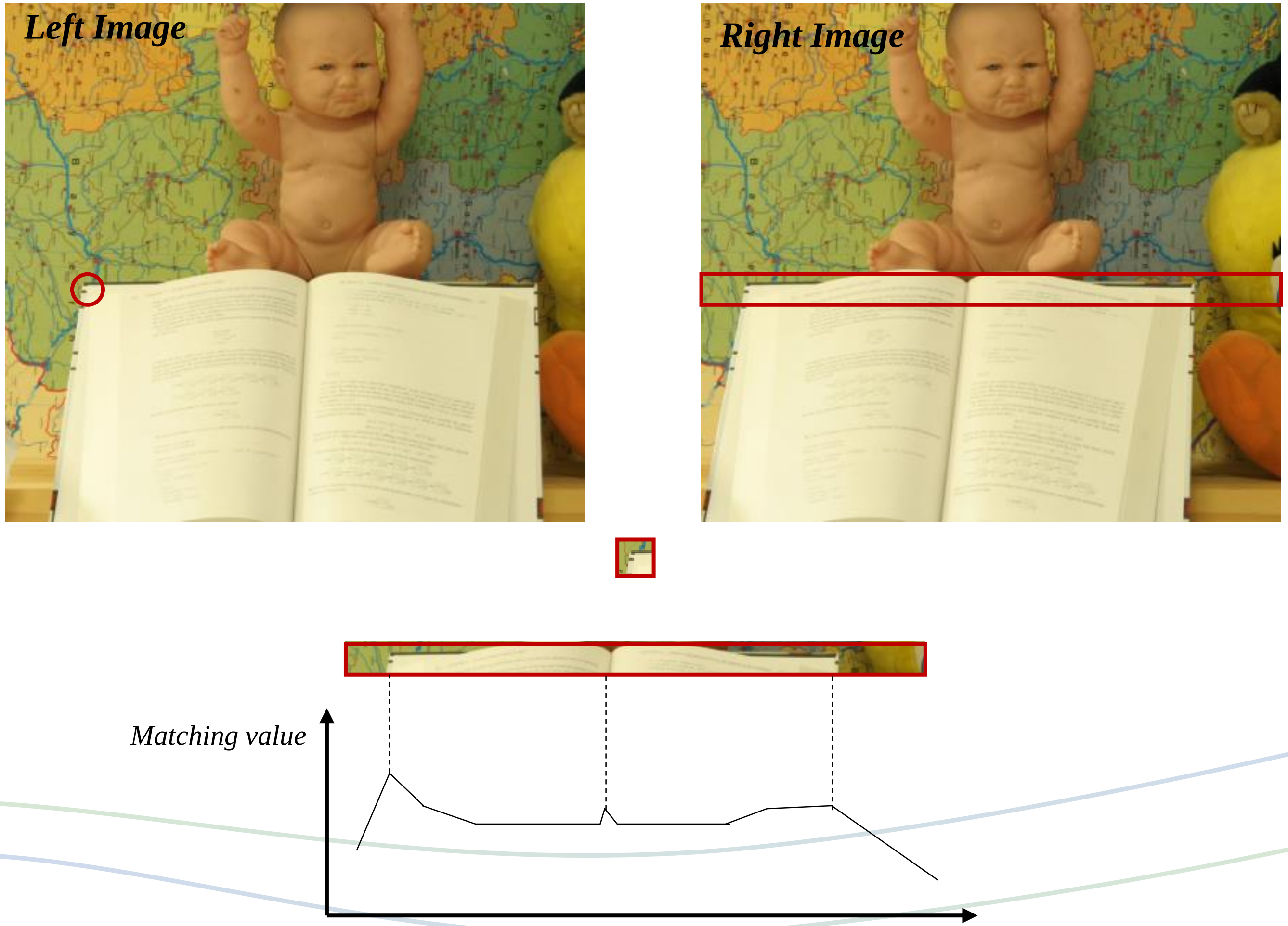
$$\therefore Z = \frac{bf}{x_l - x_r}$$

➡ We need to disparity information

Stereo Matching

❖ General idea

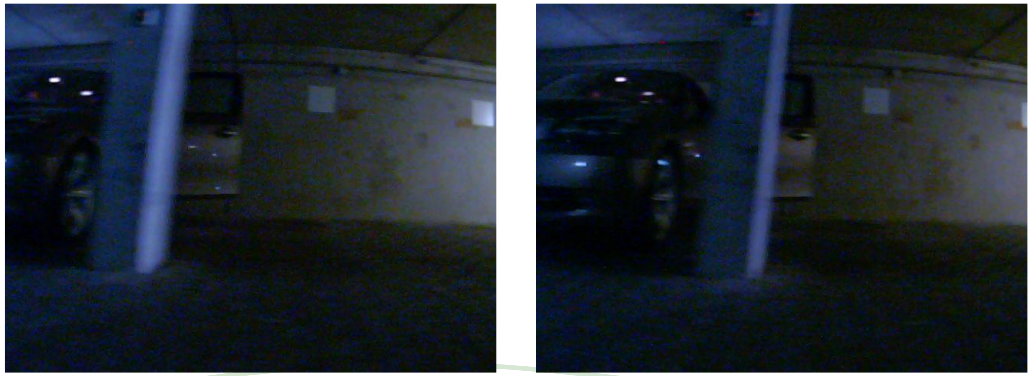
: Match along the epipolar line and Find best matching value



Stereo Matching

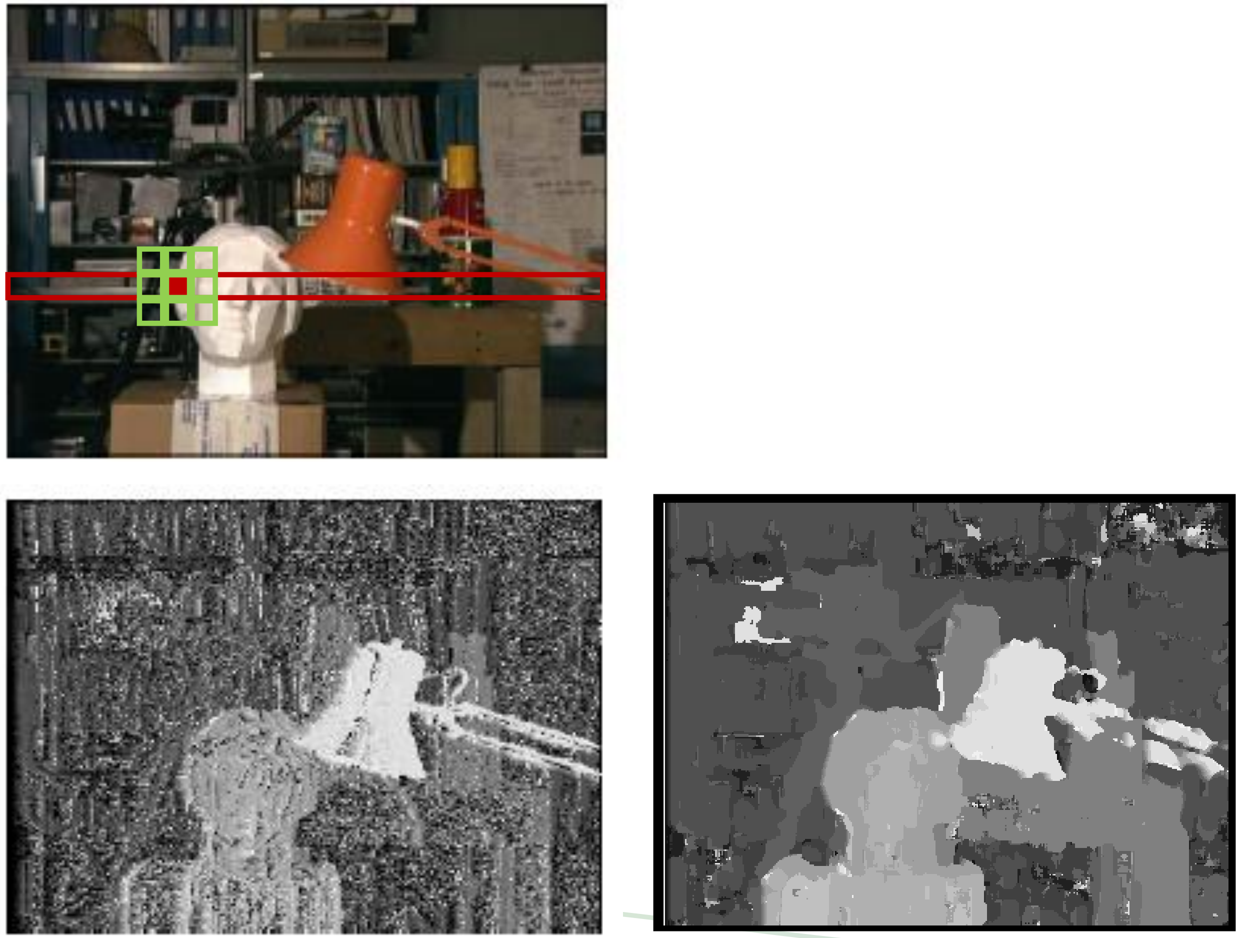
❖ Correspondence problem

- Noise
- Low-texture region
- Occlusion
- Depth-discontinuity



Stereo Matching

❖ Local method



SSD (Sum of Squared Difference)

$$SSD_{MN}(x,y,d)=\sum_{y=1}^{M}\sum_{x=1}^{N}[I_l(x,y,)-I_r(x-d,y)]^2$$

SAD (Sum of Absolute Difference)

$$SAD_{MN}(x,y,d)=\sum_{y=1}^{M}\sum_{x=1}^{N}|I_l(x,y,)-I_r(x-d,y)|$$

MAE(Mean Absolute Error)

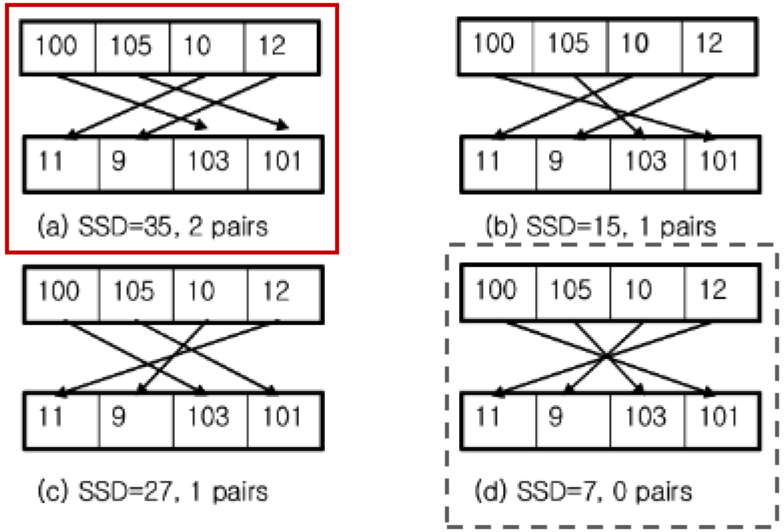
$$MAE_{MN}(x,y,d)=\frac{1}{M\times N}\sum_{y=1}^{M}\sum_{x=1}^{N}|I_l(x,y,)-I_r(x-d,y)|$$

Stereo Matching

Global method

: Use the energy functions

$$E(d) = E_{data}(d) + \lambda E_{smoothness}(d)$$



Implementation :

Belief Propagation(BP), Graph Cut(GC), Dynamic Programing(DP)…

Belief Propagation

❖ MRF (Markov Random Field)

: Undirected and cyclic

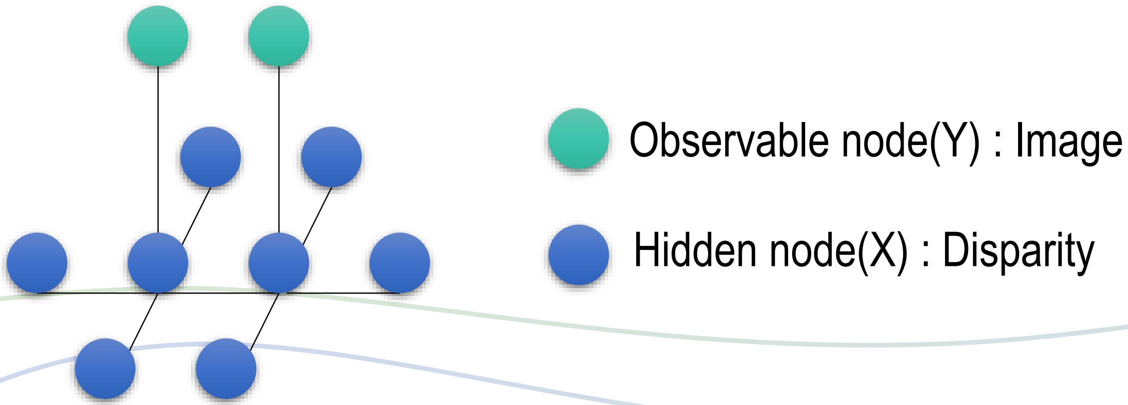
1) Positivity

$$P(f) > 0, \forall f$$

2) Markovianity

$$P(f_p \mid f_{P-\{p\}}) = P(f_p \mid f_{N_p})$$

❖ HMM (Hidden Markov Model)



Belief Propagation

❖ Goal

: Computes **marginal probability** of hidden nodes

❖ Attributes

: **Iterative** algorithm

: **Message passing** between neighboring hidden nodes

❖ Procedure

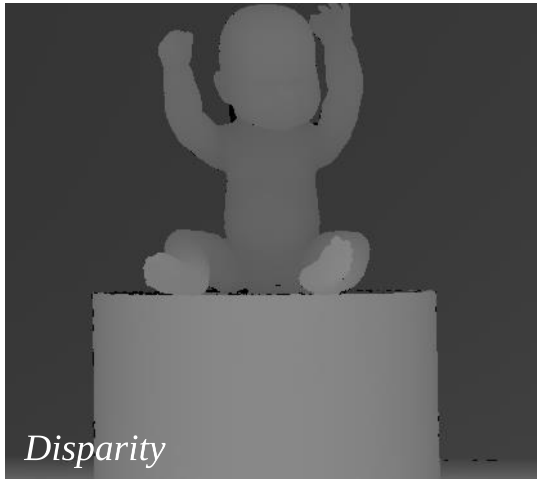
1) Select random neighboring hidden nodes x_s, x_t

2) Send message m_{st} from x_s to x_t

3) Update belief about marginal probability

Belief Propagation

❖ Probabilistic Stereo Model



likelihood

posterior

prior (Hypothesis)

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

evidence (Data)

Belief Propagation

❖ Likelihood

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

Matching cost function $F(s, d_s) = | I_l(i, j) - I_r(i - d_s, j) |$

$$P(Y \mid X) \propto \prod_s \exp(-F(s, d_s))$$

d_s: disparity candidate at pixel s

Example

20	25	117	135	150

Left Image(Ref.)

118	134	22	20	15

Right Image

Disparity(x_s)	d=0	d=1	d=2	d=3	d=4
Value(y_s)	135	130	128	16	32

Belief Propagation

❖ Prior

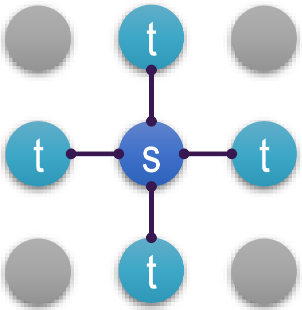
$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

Constraint function $V(d_s, d_t) = |d_s - d_t|$

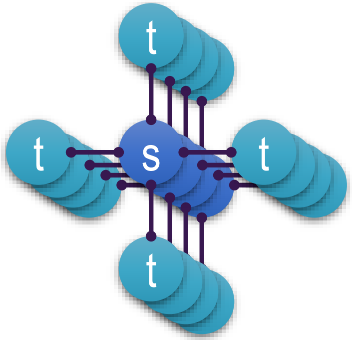
$$P(X) \propto \prod_{s} \prod_{t \in N(s)} \exp(-V(d_s, d_t))$$

d_s: disparity candidate at pixel s

d_t: disparity candidate at pixel t



Example



Belief Propagation

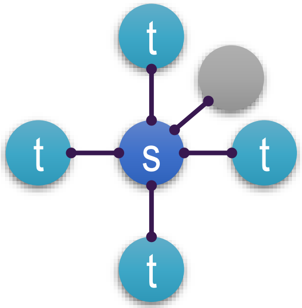
Final model

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

$$\propto P(Y \mid X)P(X)$$

$$\propto \prod_s \exp(-F(s, d_s)) \prod_s \prod_{t \in N(s)} \exp(-V(d_s, d_t))$$

$$= \prod_s \psi_s(x_s, y_s) \prod_s \prod_{t \in N(s)} \psi_{st}(x_s, x_t)$$



ψ_s : local evidence for node x_s

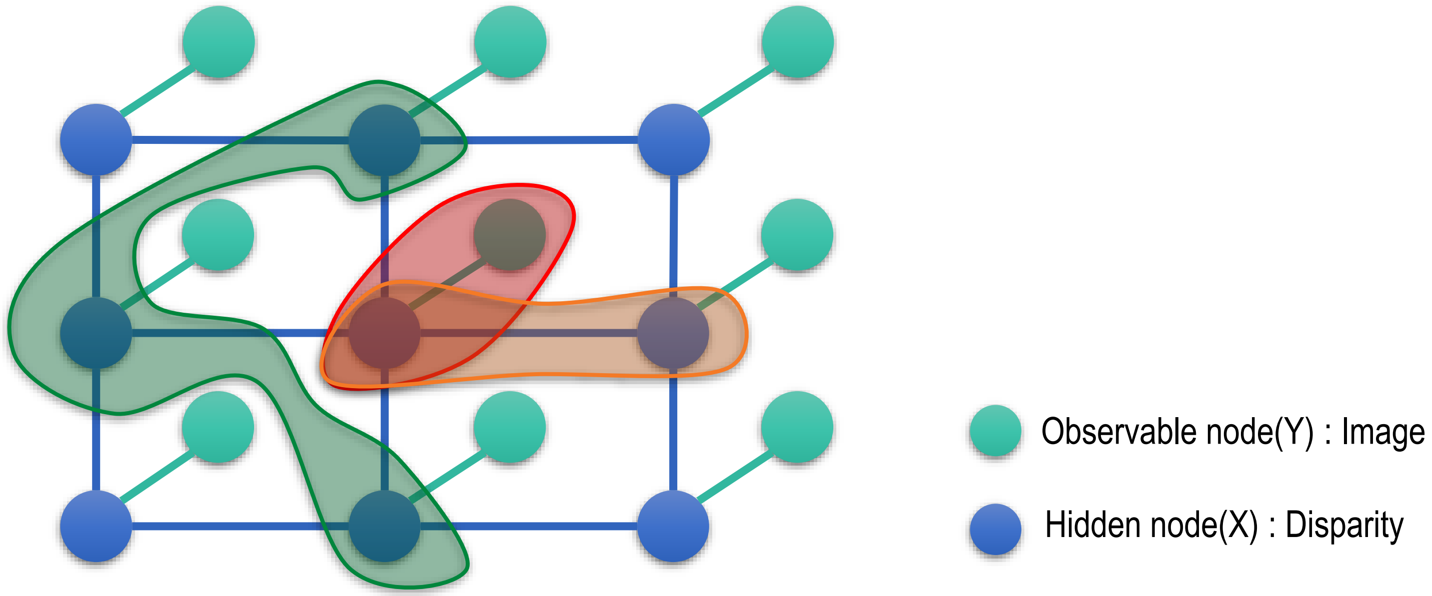
ψ_{st} : compatibility between nodes x_s and y_t

MAP ➡ maximize marginal probability (maximize belief)

Belief Propagation

❖ Message Passing

: Message m_{st} from x_s to x_t



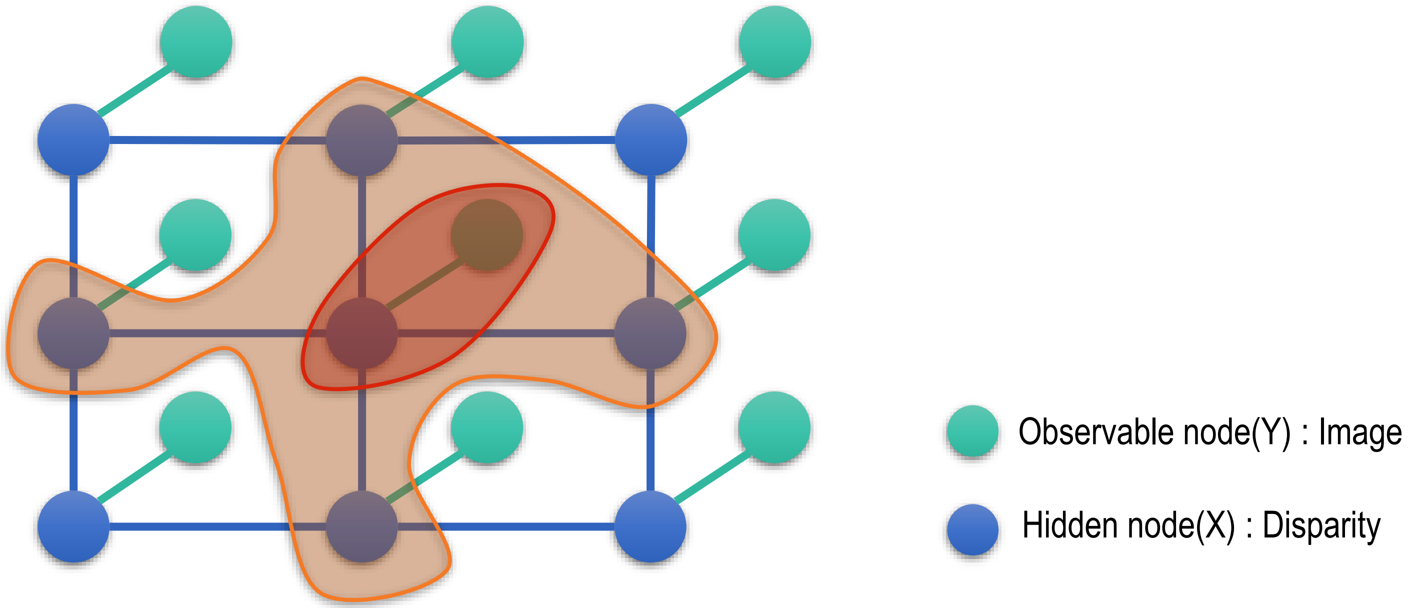
$$m_{st}(x_t) = \kappa \max_{(xs)} [\psi_s(x_s, y_s)\, \psi_{st}(x_s, x_t) \prod_{xk \in N(s)\backslash t} m_{ks}(x_s)]$$

$m_s(x_s)$: local evidence

Belief Propagation

❖ Belief Update

: Belief $b(x_s)$



$$b_s(x_s) = \kappa\, \psi_s(x_s, y_s) \prod_{xk \in N(s)} m_{ks}(x_s)$$

Belief Propagation

- Implementation of message, belief and disparity

$$m_{st}^{i+1}(x_t) = \kappa \max_{x_s}\left[\psi_s(x_s, y_s)\psi_{st}(x_s, x_t)\prod_{x_k \in N(x_s)\backslash x_t} m_{ks}^{i}(x_s)\right]$$

$$b_s(x_s) = \kappa\psi_s(x_s, y_s)\prod_{x_k \in N(x_s)\backslash x_t} m_{ks}(x_s)$$

$$d_s^{MAP} = \arg\max_{x_k} b_s(x_k)$$

take the negative logarithm of each equation

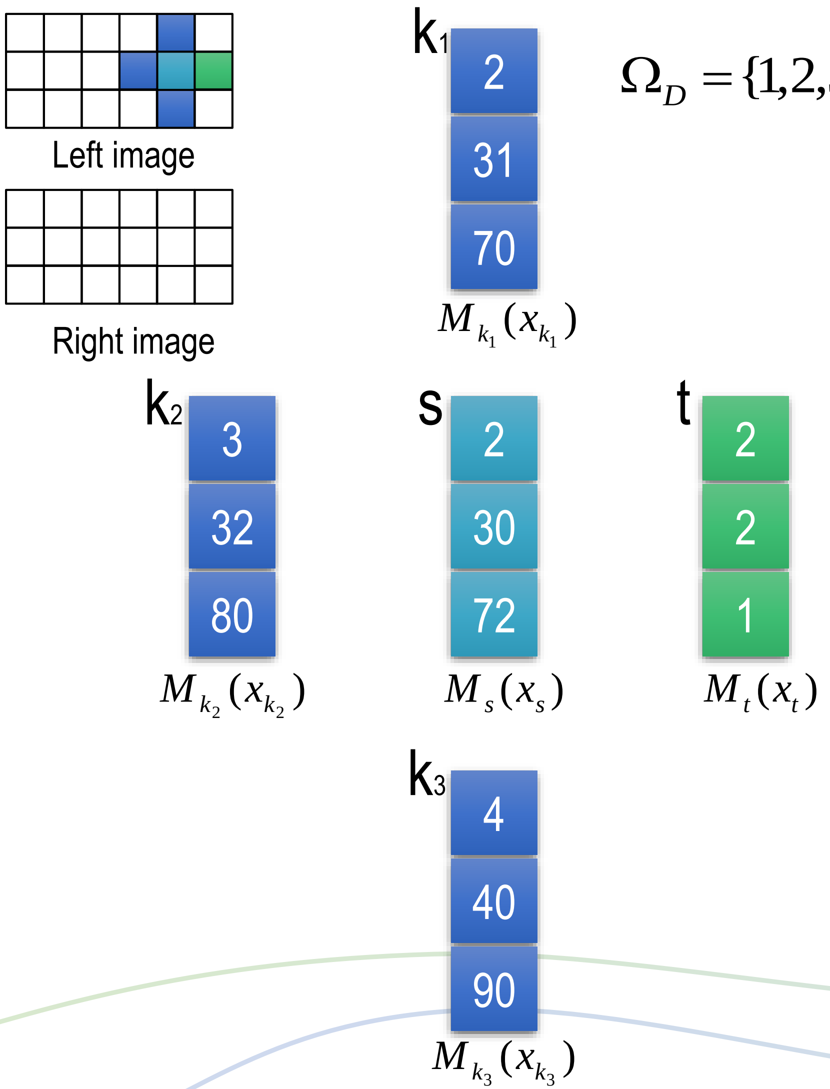
$$M_{ij}^{t+1}(x_j) = c\min_{x_i}\left[M_i(x_i) + \phi_c(x_i, x_j) + \sum_{x_k \in N(x_i)\backslash x_j} M_{ki}^{t}(x_i)\right]$$

$$B_i(x_i) = c\left[M_i(x_i) + \sum_{x_k \in N(x_i)\backslash x_j} M_{ki}(x_i)\right]$$

$$x_s^{MAP} = \arg\min_{x_k} B_s(x_k)$$

Belief Propagation

❖ Example



$$\Omega_D = \{1,2,3\}$$

$$M_{st}^{i+1}(x_t) = c \min_{x_s} \left[M_s(x_s) + \phi_c(x_s, x_t) + \sum_{x_k \in N(x_s) \backslash x_t} M_{ks}^{i}(x_s) \right]$$

1) Initialize

$$M_{st}^{0}(x_t) = [0 \quad 0 \quad 0]^T$$

$$M_{k_1s}^{0}(x_s) = M_{k_2s}^{0}(x_s) = M_{k_3s}^{0}(x_s) = [0 \quad 0 \quad 0]^T$$

2) Update

$$M_{st}^{1}(x_t = 1)$$

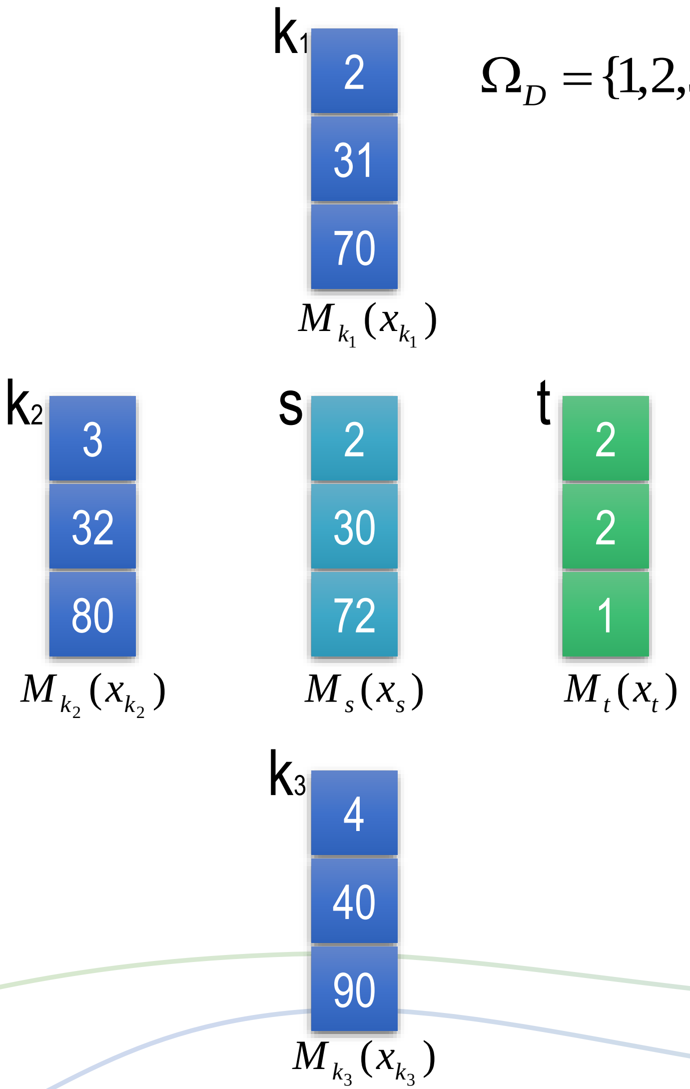
$$= \min \begin{pmatrix} (2+0+M_{k_1s}^{0}(x_s=1)+M_{k_2s}^{0}(x_s=1)+M_{k_3s}^{0}(x_s=1)), \\ (30+1+M_{k_1s}^{0}(x_s=2)+M_{k_2s}^{0}(x_s=2)+M_{k_3s}^{0}(x_s=2)), \\ (72+2+M_{k_1s}^{0}(x_s=3)+M_{k_2s}^{0}(x_s=3)+M_{k_3s}^{0}(x_s=3)) \end{pmatrix} = 2$$

$$M_{st}^{1}(x_t = 2)$$

$$= \min \begin{pmatrix} (2+1+M_{k_1s}^{0}(x_s=1)+M_{k_2s}^{0}(x_s=1)+M_{k_3s}^{0}(x_s=1)), \\ (30+0+M_{k_1s}^{0}(x_s=2)+M_{k_2s}^{0}(x_s=2)+M_{k_3s}^{0}(x_s=2)), \\ (72+1+M_{k_1s}^{0}(x_s=3)+M_{k_2s}^{0}(x_s=3)+M_{k_3s}^{0}(x_s=3)) \end{pmatrix} = 3$$

Belief Propagation

- **Example**



$$\Omega_D = \{1,2,3\}$$

$$M_{st}^{i+1}(x_t) = c \min_{x_s}\left[M_s(x_s) + \phi_c(x_s, x_t) + \sum_{x_k \in N(x_s)\backslash x_t} M_{ks}^{i}(x_s) \right]$$

$$M_{st}^{1}(x_t = 3)$$

$$= \min\begin{pmatrix} \left(2+2+M_{k_1s}^{0}(x_s=1)+M_{k_2s}^{0}(x_s=1)+M_{k_3s}^{0}(x_s=1)\right), \\ \left(30+1+M_{k_1s}^{0}(x_s=2)+M_{k_2s}^{0}(x_s=2)+M_{k_3s}^{0}(x_s=2)\right), \\ \left(72+0+M_{k_1s}^{0}(x_s=3)+M_{k_2s}^{0}(x_s=3)+M_{k_3s}^{0}(x_s=3)\right) \end{pmatrix} = 4$$

$$M_{st}^{1} = [2 \quad 3 \quad 4]^T$$

$$M_{k_1s}^{1}(x_s) = [2 \quad 3 \quad 4]^T$$

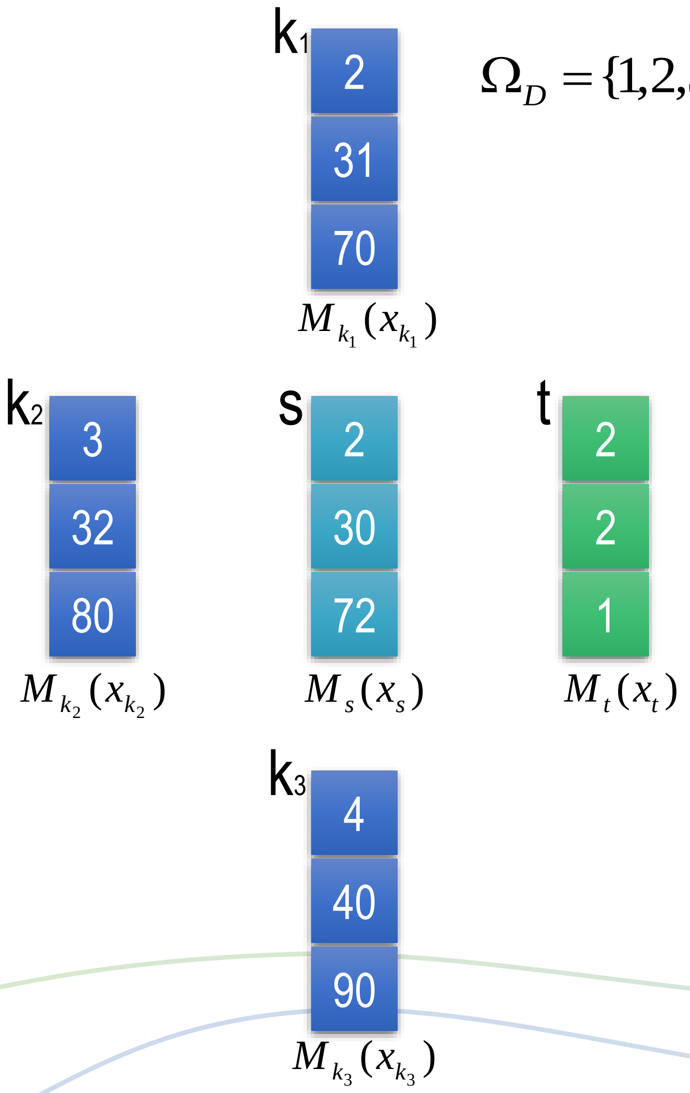
$$M_{k_2s}^{1}(x_s) = [3 \quad 4 \quad 5]^T$$

$$M_{k_3s}^{1}(x_s) = [4 \quad 5 \quad 6]^T$$

$$\Rightarrow \quad M_{st}^{2}(x_t) = [11 \quad 12 \quad 13]^T$$

Belief Propagation

- **Example**



$$\Omega_D = \{1,2,3\}$$

$$B_s(x_s) = c\left[M_s(x_s) + \sum_{x_k \in N(x_s)\setminus x_t} M_{ks}(x_s) \right]$$

$$x_s^{MAP} = \arg\min_{x_k} B_s(x_k)$$

3) Calculate Belief

$$B_t(x_t = 1) = 2 + 11 = 13$$

$$B_t(x_t = 2) = 2 + 12 = 14$$

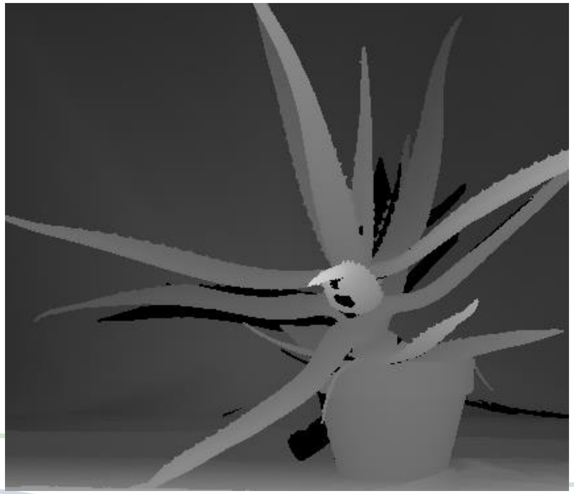
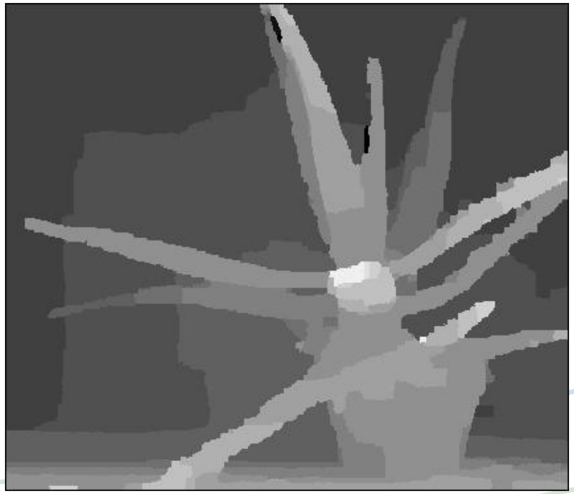
$$B_t(x_t = 3) = 1 + 13 = 14$$

$$B_t(x_t) = [13 \quad 14 \quad 14]^T, \qquad x_t^{MAP} = \arg\min_{x_k} B_t(x_k) = 1$$

Experimental Results

- Aloe*



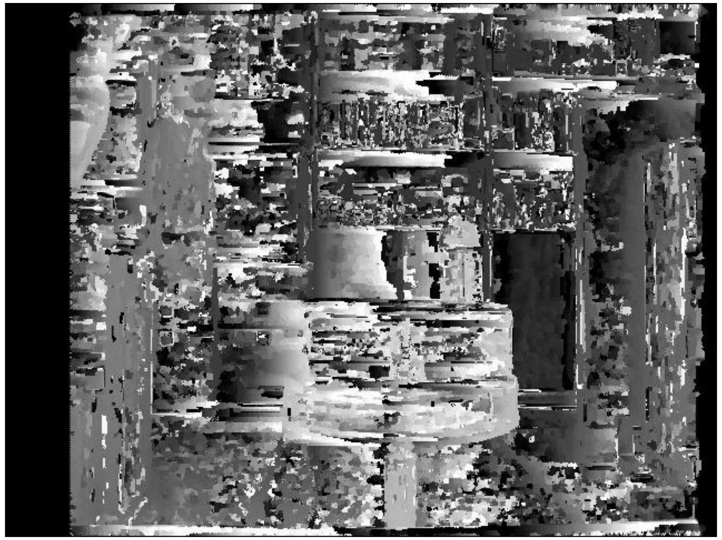



** middlebury 2006 stereo dataset*

Experimental Results

- Lab

i=1



i=5

i=20

Q & A

Thank You!!